1 Intorduction

Over the last 15 years our group at the University of Michigan has been developing a general use global MHD code, BATS-R-US, and the Space Weather Modeling Framework (SWMF) that couples domain models extending from the Sun to planetary upper atmospheres and ionospheres. BATS-R-US and the SWMF have been extensively used to simulate a broad range of space science phenomena. Still there are many unmet challenges. There is a need to go beyond ideal MHD. In the previous AISRP we have developed a new parallel implicit Hall MHD solver for the 3D block-adaptive grid used in BATS-R-US.

In the current AISR project we are further developing the BATS-R-US code. In the first year we have

- finished and published the Hall MHD scheme
- added an electron equation to the MHD equations
- added a multi-ion MHD equation module
- tested the empirical resistivity model by M. Kuznetsova

Below we will describe these developments in more detail.

1.1 Hall MHD equation

The Hall MHD equations have been successfully implemented into the BATS-R-US code. It can be used with explicit and implicit time stepping, on Cartesian and generalized block-adaptive grids and the code scales well to hundreds of processsors. The details of the algorithm and a series of tests have been published in the Journal of Computational Physics (G. Toth, Y. J. Ma, T. I. Gombosi, 2008, JCP, 227, 6967-6984).

The Hall MHD code has been used for a number of space physics runs, including the reconnection of the magnetotail of the Earth, and the space plasma environment around unmagnetized planets and moons. The magnetotail simulations showed some fast quasi-periodic plasmoid formation that is not unlike some of the observed features. The results are not conclusive, though, because it is difficult to achieve a good enough spatial resolution and still finish the run in a reasonable time. Modeling the non-magnetized planets and moons seems to be an easier task. Here the Hall effect modifies the overall solution but it is not confined to a small region. We have published a paper (3D global multi-Species Hall-MHD simulation of the Cassini T9 flyby, Y. J. Ma et al, 2007, Geophysical Research Letters, 34, L24S10). The results presented in this paper suggest that the magnetic field measured by Cassini can be better reproduced when the Hall term is included into the MHD equations.

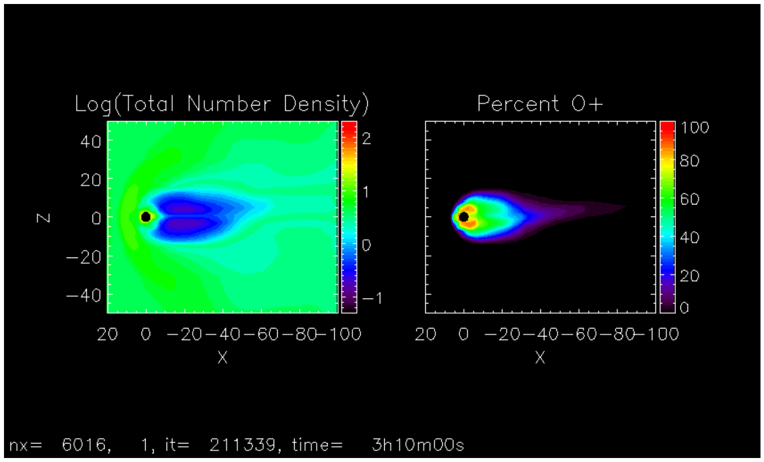

Figure 1: An example of a multifluid MHD result driven by PWOM outflow at the inner boundary and solar wind at the outer boundary. Quantities shown are the log of total number density and what percent of the number density is composed of oxygen in the noon-midnight $y = 0$ plane

1.2 Two-fluid equation

The MHD equations can now be extended with an extra equation for the electron pressure p_e:

$$\frac{\partial p_e}{\partial t} + (\mathbf{u}_e \cdot \nabla)\, p_e + \frac{5}{3} p_e \nabla \cdot \mathbf{u}_e = S(p_e) \tag{1}$$

where the electron velocity is either the same as the ion velocity or it can include the Hall velocity so that $\mathbf{u}_e = \mathbf{u} - \frac{\mathbf{j}}{/}(e\, n_e)$. The right hand side S represents arbitrary source terms.

This extra equation allows the ion and electron pressures/temperatures to be different. We plan to use this new capability to model the electron temperature in the solar corona that can be substantially different from the ion temperature, and the observations typically measure the electron temperature rather than the ion tempreature.

1.3 Multi-ion MHD equations

We have implemented a new algorithm for the multi-ion MHD equations. The momentum equation for the ion fluid s is written as

$$\frac{\partial \rho_s \mathbf{u}_s}{\partial t} + \nabla \cdot (\rho_s \mathbf{u}_s \mathbf{u}_s + I p_s) = \rho_s \frac{q_s}{m_s} (\mathbf{u}_s - \mathbf{u}_e) \times \mathbf{B} + S(\rho_s \mathbf{u}_s) \tag{2}$$

where q_s and m_s are the charge and mass of one ion, respectively, and $\mathbf{u}_e = \mathbf{u}_+ - \mathbf{j}/(e\, n_e)$. Here $\mathbf{u}_+$ is the average ion speed weighted by charge rather than mass density, e is the electron charge, and n_e is the electron number density.

The individual momentum equations cannot be written in conservation form. To obtain the correct jump conditions across shock waves, we also solve the total MHD equations in conservation form in addition to the ion equations. After each time step the individual and total ion quantities are adjusted so that they add up correctly.

We use a point-implicit evaluation for the right hand side to make the scheme stable. A first order in time version of the scheme is

$$(\rho_s \mathbf{u}_s)^{n+1} = (\rho_s \mathbf{u}_s)^n - \Delta t \nabla \cdot \mathbf{F}^n + S_M(\rho^n, \rho\mathbf{u}^{n+1}, \mathbf{B}^n) + S(\rho_s \mathbf{u}_s)^n \tag{3}$$

where $\mathbf{F}$ contains the hydrodynamic fluxes, and S_M is the right hand side of equation (2) without S. Note that the magnetic field and densities are evaluated at time level n. The multi-ion source term S_M is linearized around the n-th time step

$$S_M^{n+1} \approx S_M^n + \frac{\partial S_M}{\partial \rho\mathbf{u}} \cdot (\rho\mathbf{u}^{n+1} - \rho\mathbf{u}^n) \tag{4}$$

where $\rho\mathbf{u}$ is a vector of the individual momenta for all the ion fluids. For sake of efficiency the Jacobian $\partial S_M / \partial \rho\mathbf{u}$ is calculated analytically.

We have used the multi-fluid code to model the magnetosphere including the oxygen outflow from the ionosphere. The outflow can be specified as a constant flow with fixed densities and velocities for the two fluids, or we can use the Polar Wind Outflow Model (PWOM) to provide these quantities at the inner boundary of the magnetosphere model (BATS-R-US).

1.4 Evaluating the non-gyrotropic resistivity model

We have done systematic runs to evaluate the non-gyrotropic resistivity model. We found that the periodicity and magnitude of the saw-tooth-like oscillations depends on the density, velocity and magnetic field in the solar wind. However it will require further studies to see how much the oscillations produced by the non-gyrotropic resistivity model mimic reality.